Erratum: Observing the Coulomb shifts of ionization times in high-order harmonic generation [Phys. Rev. A 107, 063102 (2023)]

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In our article, there was an error in the numerical implementation of the Coulomb-modified equations used for the retrieval of ionization and recombination times in high-order harmonic generation from solutions of the time-dependent Schrödinger equation (TDSE) based on Eq. (7) of the article. Here we show the revised Fig. 3 with corrections in Fig. 3(b). Specifically, the violet triangles in Fig. 3(b), labeled TDSE ω-2ω, are corrected. Here we observe that, (i) as before, the return times from the TDSE align closely with the quantum orbit (QO) model, the analytical R-matrix (ARM) theory, and the adiabatic model and (ii) unlike before, the reconstructed ionization times from the TDSE deviates significantly from the ARM model and the adiabatic model. However, the overall trend caused by the Coulomb modification of the retrieval equations is still intact in the sense that the Coulomb modification moves the TDSE ionization times away from the QO model towards earlier times.

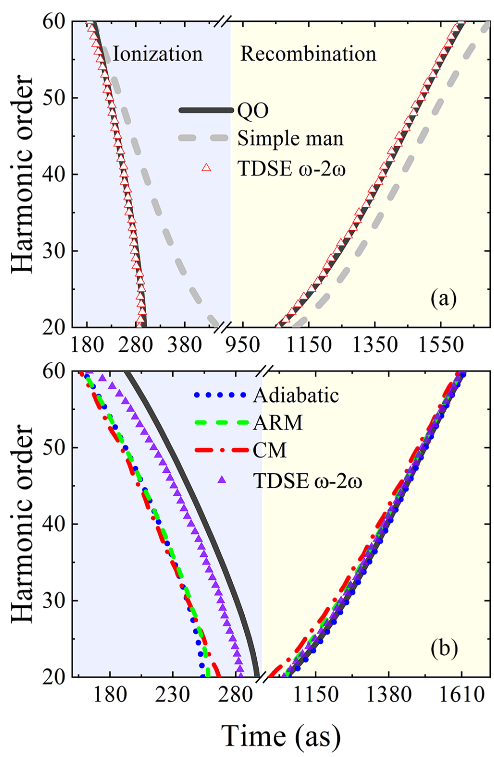

FIG. 3. Ionization and return times reconstructed from the ω-2ω TDSE results. (a) Results based on the Coulomb-free orthogonally polarized two-color (OTC) scheme shown as open red triangles and (b) results based on the Coulomb-corrected OTC scheme shown as closed violet triangles. The black solid lines represent the real parts of the times from the Coulomb-free QO model. The light-gray dashed line shows the times from the simple man's model. Furthermore, ionization and return times from various models with Coulomb interaction are shown: the ARM theory (green dashed lines), the CM (red dash-dotted lines), and the adiabatic model (blue dotted lines). Light-blue and light-yellow shaded areas show the temporal regions of ionization and recombination, respectively.

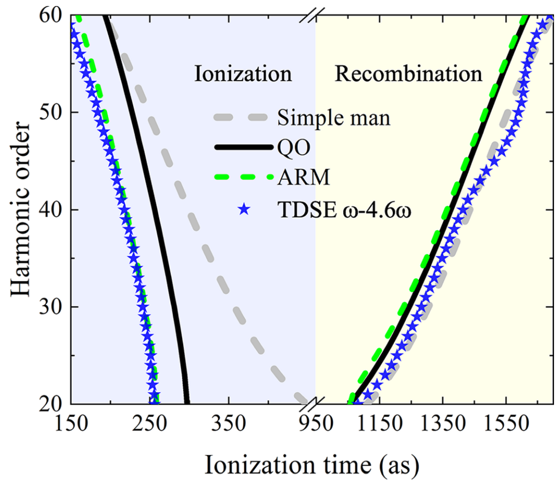

FIG. 4. Ionization and return times from the TDSE with an ω-4.6ω field (blue stars) compared to the ARM theory (green dashed lines), the QO model (black solid lines), and the simple man's model (light-gray dashed lines).

In the Coulomb-modified retrieval equations, the Coulomb-induced momentum change in the direction of the probe field is approximated as a short kick acquired by an outgoing electron while treating the probe field as static. Note that the same idea is used to derive the adiabatic model, which results in good agreement with the ARM theory and the classical model (CM) for the ionization time [see Fig. 3(b)]. This suggests that the real Coulomb kick derived with a static treatment of the ionizing field is not a good approximation when used in combination with the complex-time retrieval equations. In contrast, the adiabatic model involves only real times, so the combination with the real Coulomb kick appears consistent and produces accurate results.

As a result of the aforementioned error, Fig. 4, showing results for higher probe frequency, requires updating as well. Notably, in this case the TDSE ionization times remain practically unaffected, while the reconstructed return times are changed and no longer match the ARM times. These deviations point to a problem of high-frequency probing that is specific to the return times and which requires future investigation.

The conclusions of the article are weakened as follows: (i) The real Coulomb kick proposed as an approach to include the Coulomb force in the time-retrieval method moves the retrieved ionization times into the right direction, but the method is not quantitatively accurate, and (ii) the retrieved return times acquire an unwanted shift to later times when high-frequency probing is used. In the future, it may be beneficial to improve the retrieval method by incorporating a Coulomb modification that goes beyond the static field approximation. The findings of the article are unchanged as far as the following points are concerned: (i) The Coulomb-modified method shows no shortcomings in retrieving the return times in low-frequency probing and the ionization times in high-frequency probing and (ii) the Coulomb kick performs well in the real-time adiabatic model in the sense that the results agree well with the ARM model.

We are indebted to Mohammad Monfared for pointing out the implementation error to us.

DATA AVAILABILITY

The data that support the findings of this article are not publicly available. The data are available from the authors upon reasonable request.